Mini-Project 1

AE4802 Configuration Aerodynamics

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1. Using OpenVSP's Parasite Drag analysis tool, I found that the total zero-lift parasite drag Cd_o at 36,000 ft and Mach 0.83 was:

Cd_o = 0.01290

2.

a. In order to find the trim conditions at 36,000 ft and Mach 0.83, I used the equations of motions to solve for the total coefficient of lift in order for lift to equal weight of the aircraft. Lift for our trimmed aircraft is the sum of the lift caused by the wing offset by the lift caused by the stabilizer (the rotating tail).

I used VSPAero to find how C_l changed with angle of attack as well as well as tail angle, substituted partial derivatives with values from VSPAero taken by changing either the angle of attack and keeping the tail angle constant or changing the tail angle while keeping the angle of attach constant. I did the same to find a constant value for the coefficient of moment about the y-axis.

The substituted values are here:

```
dCl_da = 0.116729012; %partial Cl wrt angle of attack
dCl_dit = 0.01412273; %partial Cl wrt tail angle
dCmy_da = -0.054477697; %partial Cmy wrt angle of attack
dCmy_dit = -0.05596845; %partial Cmy wrt tail angle
```

VSPAero returned a value of Cl_o = 0.3277, which I chose to be at an angle of attack of zero and tail angle of zero.

In order to solve for the trim angles and angle of attack, I set up a system of two equations:

1) The total lift coefficient is the sum of the partial derivative of C_l with respect to angle of attack, the partial derivative of C_l with respect to the tail angle, and Clo.
2) The total moment coefficient is the sum of the partial derivative of C_{my} with respect to angle of attack, the partial derivative of C_{my} with respect to the tail angle, and C_{myo}, which VSPAero calculated at -6.37×10^{-2}

```
Cl = W/(q*s);
Cmy0 = -6.37E-02;

eq1 = Cl == dCl_da*a_trim + dCl_dit*it_trim + Cl0;
eq2 = 0 == dCmy_da*a_trim + dCmy_dit*it_trim + Cmy0;
```

I calculated the total lift coefficient by dividing the weight of the aircraft by dynamic pressure multiplied by the wing reference area. I set C_{my} equal to zero because this would be ideal for a trim condition.

Using MATLAB, I was able to find my trim conditions:

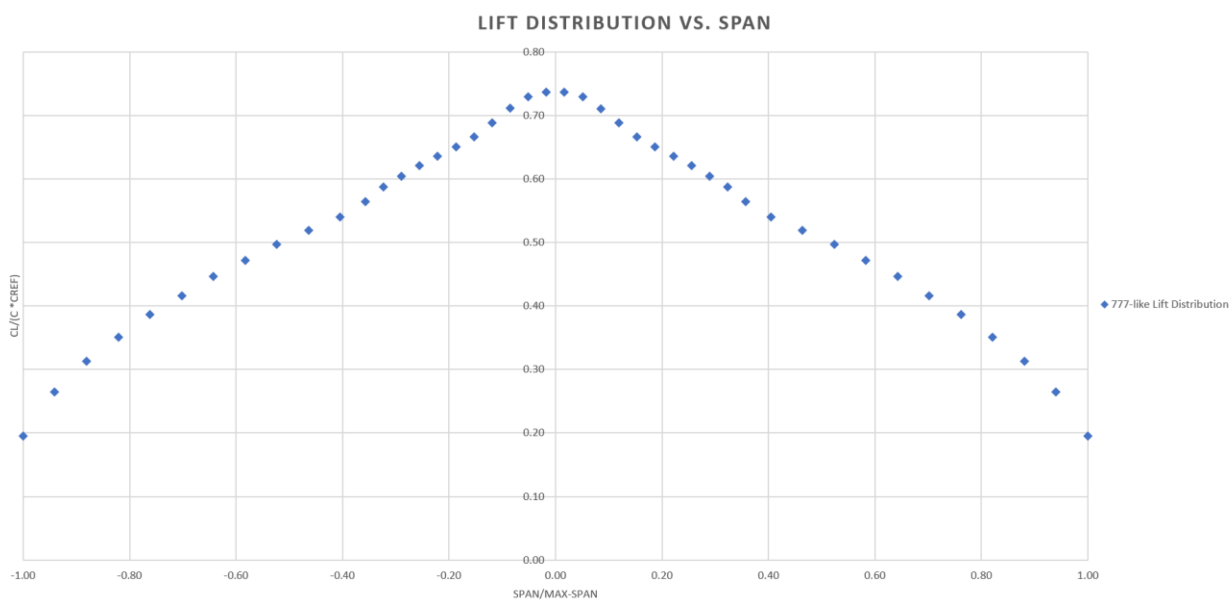
```
eqns = [eq1 eq2];
eq_solve = solve(eqns, [a_trim it_trim]);

a_trim = double(eq_solve.a_trim);
it_trim = double(eq_solve.it_trim);

Cl_trim = dCl_da*a_trim + dCl_dit*it_trim + Cl0;
```

Cl_{trim}: 0.4685

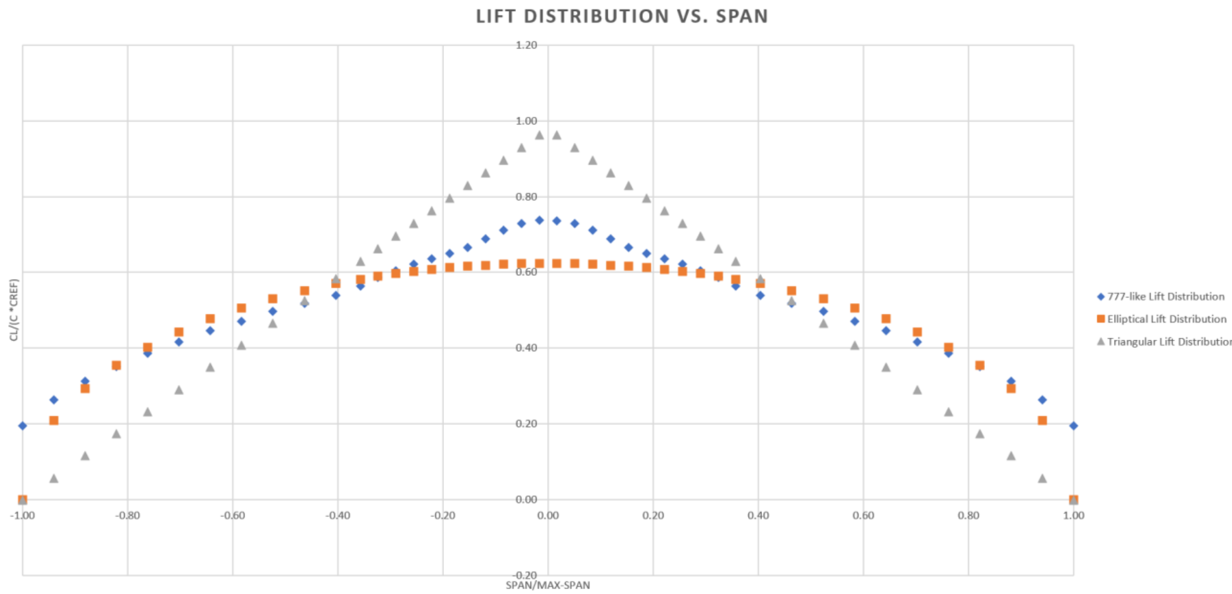
α_{trim}: 1.5227 degrees

$i_{t,trim}$: -2.6203 degrees

b. Below is a plot of the lift distribution on the wing of the 777-like aircraft at trim conditions at 36,000 ft and Mach of 0.83:



The shape of the lift distribution on the wings at my $C_{l,trim}$ of **0.4685** is rounded out at the wing tips and then more steep towards the center. It seems like it is a blend of triangular and elliptical lift distribution, leaning more towards triangular.



When plotted against the elliptical distribution, it's easy to see the triangular "bump" starting at around the quarter-span. From the quarter-span onwards, the shape becomes more elliptical. I'd say that the 777-like lift distribution is more elliptical than triangular.

Lift over the wing span is a function of the chord length as well as coefficient of lift of each discreet airfoil along the span. Therefore in order to change the shape of a wing (assuming the airfoil shape remains the same), we can either vary its chord length, change the coefficient of lift on a given section by twisting the section to obtain a higher angle of attack, or some combination of both.

In order to achieve a more triangular lift distribution by twisting the wing, it would be necessary to twist the section of each wing from the fuselage to the quarter-span to a higher angle of attack and/or twist the section from the quarter-span outwards to a lower angle of attack.

In order to achieve a more elliptical lift distribution by twisting the wing, the opposite is necessary: twist the section closer to the fuselage to lower the angle of attack.

c. The C_{Di} at this trim point is:

C_{Di} = 0.009817

3. In order to solve for the wave drag coefficient, I used the Korn equation and Lock's 4^{th} power method. The MATLAB code below shows the steps I took, which includes taking some geometry of the wing portions and calculating other geometry from OpenVSP:

```
k = 0.95; %supercritical airfoil
root1 = 50; %feet
tip1 = 30; %feet
root2 = tip1; %feet
tip2 = 10; %feet
cmid1 = (root1+tip1)/2;
cmid2 = (root2 + tip2)/2;
sweep1 = 32*pi/180;
sweep2 = 30*pi/180;
t_c1 = 0.0998;
t_c2 = t_c1;
span1 = 40/2; %feet point along span at which we are analysing Cl1
span2 = span1 + 70/2; %feet. Point along span at which we are analysing Cl2
Cl1 = 0.49; %from VSPAero
Cl2 = 0.59; %from VSPAero
```

Below shows the use of the Korn equation and Lock's 4^{th} power method used to find the wave drag coefficients for both sections of the wing.

```
syms Mdd1 Mdd2

Korn1 = Mdd1*cos(sweep1) + Cl1/(10*(cos(sweep1)^2)) + t_c2/(cos(sweep1)) == k;
Mdd1 = double(solve(Korn1,Mdd1));
Mcr1 = Mdd1 - (0.1/80)^(1/3);
Korn2 = Mdd2*cos(sweep2) + Cl2/(10*(cos(sweep2)^2)) + t_c2/(cos(sweep2)) == k;
Mdd2 = double(solve(Korn2,Mdd2));
Mcr2 = Mdd2 - (0.1/80)^(1/3);

%Lock's 4th Power method

Cdw1 = 20*((0.83 - Mcr1) ^ 4);
Cdw2 = 20*((0.83 - Mcr2) ^ 4);
```

Below is a table of values found using both VSPAero, OpenVSP's geometry, and MATLAB:

	Strip 1	Strip 2
thickness-to-cord ratio	0.0998	0.0998
half-chord sweep angle (rad)	0.5585	0.5236
lift coefficient	0.49	0.59
airfoil technology factor	0.95	0.95
drag divergence mach number	0.9011	0.8731
critical mach number	0.7934	0.7653
wave drag coefficient	3.59E-05	3.50E-04

Using an equation for area-weighted average, I was able to find an averaged total wave drag coefficient.

```
%Area-weighted average

S1 = 1600; %sq ft
S2 = 1400; %sq ft
Cdw_tot = (Cdw1*S1 + Cdw2*S2)/(S1 + S2);
```

The wave drag coefficient at the flight conditions stated for this project is:

C_{Dw} = 0.0001823

4. The total drag coefficient, C_D is a sum of C_{Do}, C_{Di}, and C_{Dw}:

C_D = 0.02290

The lift to drag ratio (L/D) of the airplane is:

L = 750,000 lbs or about 3.33×10^3 kN

D = 0.16×10^3 kN

(Drag was calculated by multiplying the drag coefficient with dynamic pressure and the wing reference area.)

L/D = 20.5

The lift-to-drag ratio I found was 20.5, which is higher than the lift-to-drag ratio of 19.3, which is what I found by searching B777 specifications online. There are probably more factors contributing to drag that this exercise did not account for. For example, the excrescence drag was set at an arbitrary 20%.

I do think that the number seems reasonable. The 777 has incredible weight and would need to generate a lot of lift to compensate.